B.Sc. Part II (Hons.) 4th Paper

DIFFERENTIAL EQUATIONS

EXACT DIFF. EQUNS. (EDE)

Definition The diff. eqn $M\,dx + N\,dy = 0$ is EDE if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Solution

$\int M\,dx$ (treat y as constant) $+ \int$[all those terms of N which do not contain x] dy = Constant

Integrating factor (IF) A factor which when multiplied to a non-exact diff. eqn makes it exact is known as IF.

Methods By inspection

Q I $a(x\,dy + 2y\,dx) = xy\,dy$

Soln. The equation is not exact. dividing both sides by xy.

$\Rightarrow a\left(\frac{dy}{y} + \frac{2\,dx}{x}\right) = dy$. integrating, we get

$\Rightarrow a\log y + 2a\log x - y = \log k$

~~Q.2~~

Method 2

If $Mx + Ny \neq 0$ for a homogeneous equation $M dx + N dy = 0$ then $\frac{1}{Mx+Ny} = IF$.

Example

Solve $(x+y)dy + (x-y)dx = 0$

Soln

$(x-y)dx + (x+y)dy = 0$ is not exact as $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Here $M = x-y$ and $N = x+y$ are homogeneous.

and $Mx + Ny = x(x-\cancel{y}) + y(\cancel{x}+y) = x^2 + y^2 \neq 0$.

So $\frac{1}{Mx+Ny} = \frac{1}{x^2+y^2}$ is the IF.

Multiplying the given eqn. with IF, we get

$$\frac{x-y}{x^2+y^2}dx + \frac{x+y}{x^2+y^2}dy = 0$$

Re-arranging them, we get

$$\Rightarrow \frac{x\,dy - y\,dx}{x^2+y^2} + \frac{x\,dx + y\,dy}{x^2+y^2} = 0$$

Integrating we get

$$\Rightarrow \tan^{-1}\frac{y}{x} + \frac{1}{2}\log(x^2+y^2) = \log k$$